Calculating Optimum Product Combinations from Standing Trees

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FRONTISPIECE.—This 50-inch-diameter white oak may contain either veneer, staves, or lumber. The market selected will determine the tree's value.

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Two statements are repeated time and again in forestry circles: Wood is the most versatile of nature's raw materials; and maximum returns can be had only by selling timber for its "highest" use. These statements raise some "knotty" questions. What portions of a hickory tree should be used for handle stock and which for lumber? Should we make all possible lumber from a log or leave a squared cant in the center for chipping; and if so, how large a cant? What parts of a white oak tree should be used for lumber, staves, or veneer to yield the maximum returns?

The first step in answering these questions is to determine the volume relations among the various potential products. For example, how would the board-foot yield of a given tree or log compare with the chord-foot yield?

White oak, which has a potential for several product combinations, can be used to establish volume relations between two products and to show how such relations can be used in valuation. In this report, a number of assumptions are made which permit comparative analysis. Before actual applications are made, on-the-ground field checks of volume and quality of trees to be sold should be made.

Developing Volumes for Product Alternatives

We can develop volume relations empirically, using statistical techniques, or we can do this theoretically. The theoretical approach was chosen for white oak from which lumber, staves, or veneer can be recovered. Assumptions were made about variables such as tree and log dimensions, product specifications, and product volumes. Gross volume, for example, was used for the initial calculations. Later some assumptions were relaxed when the relations were applied to valuation problems.

Assumptions About Trees

For this analysis white oak tree size was assumed to range from 16 to 30 inches d.b.h., and from 16 to 64 feet in merchantable length (1 to 4 logs). Rational form classes chosen for all diameters were based on form classes suggested as appropriate in the literature and on field data. From 79 for a 16-inch tree to 86 for a 30-inch tree, form class increased once for each 2-inch diameter class. D.b.h. inside bark was determined using the k factor described by Meyer.[1]

A field study of 86 trees indicated that a k factor of 0.92 was satisfactory. For butt logs, a straight-line taper from the top of the log to d.b.h. inside bark was assumed. From the taper, inside-bark diameters to the nearest 0.1 inch were calcu-

[1]Meyer, H. Arthur. Forest mensuration. 357 pp., illus. State College, Pa. 1953.

lated for each 3.25-foot section (stave-bolt length). Upper log tapers inside bark were calculated from table 2 in Mesavage and Girard.[2]

Assumptions About Products

Log and veneer-bolt specifications for our example are defined by the International ¼-inch rule and by the Doyle rule. Board-foot volumes, calculated from the scaling diameter at the end of each 16-foot log to a 10-inch top diameter, were taken from Mesavage and Girard. Sections split from the 39-inch stave bolts had to include a 5-inch square of heartwood (fig. 1). There are 5 bolts per 16-foot log. Heartwood diameter was estimated by deducting the average double sapwood thickness, 2.2 inches as determined from field measurements, from the inside-bark diameters. Sections split to yield the 5-inch square shown in the following tabulation, are in accord with unpublished calculations.[3]

Heartwood diameter (inches)	*Fractional split*	*Chord length of split section (inches)*
12	½	12
14	⅓	12⅛
16	¼	11½
18	⅕	10¾
20	⅙	10
22	⅙	11
24	⅛	9⅛
26	⅛	9⅞
28	⅑	9⅝
30	⅑	10¼

To obtain stave volume for bolts, chord lengths of the split sections were summed. To obtain stave volume for logs, chord feet of the bolts were summed. Logs required at least a 14-inch scaling diameter to yield staves.

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[2]Mesavage, Clement and Girard, James W. Tables for estimating board-foot volume of timber. 94 pp. U.S. Forest Serv. 1946.

[3]From unpublished tables compiled in 1958 by George Gleim.

460 board feet, while 27.1 chord feet could be recovered from the upper two logs. Clearly, more product combinations can be calculated as alternatives for this one tree. But more board feet are sacrificed for every chord foot obtained from some combinations than from others (table 2). More lumber in board feet would also be sacrificed in the complete stave utilization of some trees than others. By the International ¼-inch rule, only 10.4 board feet could be recovered for every chord foot in a 1-log, 18-inch tree, while 34.5 board feet could be recovered per chord foot from a 3-log, 16-inch tree. Stave utilization is likely to be more profitable at the expense of only 10.4 board feet per chord foot than at the expense of 34.5 board feet.

TABLE 2.— Board feet per chord foot for white oak trees (gross scale)

D.b.h. (inches)	1-log trees		2-log trees		3-log trees		4-log trees	
	Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle
16	16.6	11.4	28.3	18.3	34.5	22.2		
18	10.4	7.8	15.7	11.3	18.8	13.1	21.3	15.9
20	10.6	8.5	10.8	8.4	13.4	10.2	14.1	10.4
22	11.5	9.7	11.3	9.2	11.2	9.0	12.7	10.1
24	12.4	11.0	12.0	10.3	11.7	10.0	12.0	10.0
26	13.4	12.2	12.8	11.5	12.6	11.1	12.2	10.7
28	14.6	13.7	13.9	12.9	13.6	12.4	13.2	11.9
30	15.8	15.2	15.0	14.3	14.6	13.7	14.2	13.2

The number of board feet that can be recovered as an alternative to recovery of 1 chord foot will be called the board-foot/chord-foot ratio in this study. It can be computed not only for whole trees but for individual logs (table 3). Different ratios are associated with different logs. For example, consider the ratios for a 3-log, 20-inch tree:

$$\text{Log 1} \quad \frac{185 \text{ board feet}}{17.5 \text{ chord feet}} = 10.6 \text{ board feet per chord foot}$$

$$\text{Log 2} \quad \frac{146 \text{ board feet}}{13.3 \text{ chord feet}} = 11.0 \text{ board feet per chord foot}$$

$$\text{Log 3} \quad \frac{108 \text{ board feet}}{2.0 \text{ chord feet}} = 54.0 \text{ board feet per chord foot}$$

Only part of log 3 is usable for staves, resulting in the extreme ratio of 54 board feet per chord foot.

TABLE 3.—Board feet per chord foot for white oak logs by log position, tree diameter, and length (gross scale)

D.b.h. (inches)	Log no.	1-log trees		2-log trees		3-log trees		4-log trees	
		Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle	Int. 1/4 inch	Doyle
16	1	16.6	11.4	16.6	11.4	16.6	11.4		
18	1	10.4	7.8	10.4	7.8	10.4	7.8	10.4	7.8
	2			52.0	35.5	27.3	17.3	28.5	18.8
20	1	10.6	8.5	10.6	8.5	10.6	8.5	10.6	8.5
	2			11.1	8.2	11.0	8.3	10.4	8.5
	3					54.0	37.0	18.6	13.1
22	1	11.5	9.7	11.5	9.7	11.5	9.7	11.5	9.7
	2			11.0	8.7	11.0	8.9	11.3	9.2
	3					11.1	8.3	10.9	8.2
	4							50.0	33.0
24	1	12.4	11.0	12.4	11.0	12.4	11.0	12.4	11.0
	2			11.4	9.5	11.8	10.0	12.1	10.3
	3					10.7	8.6	10.9	8.8
	4							12.6	9.1
26	1	13.4	12.2	13.4	12.2	13.4	12.2	13.4	12.2
	2			12.3	10.7	12.7	11.4	13.0	11.6
	3					11.4	9.5	11.5	9.7
	4							10.3	8.1
28	1	14.6	13.7	14.6	13.7	14.6	13.7	14.6	13.7
	2			13.2	11.9	13.8	12.5	14.0	12.8
	3					12.1	10.6	12.8	10.9
	4							11.0	9.0
30	1	15.8	15.2	15.8	15.2	15.8	15.2	15.8	15.2
	2			14.2	13.3	14.8	13.9	15.0	14.2
	3					13.0	11.8	13.5	12.3
	4							11.6	10.0

Checks of the Theoretical Relations

It must be reiterated that the volume relations were developed theoretically. They mean little unless the white oak board-foot/chord-foot ratios developed actually prove themselves in practice. Since the performance of board-foot volume tables is well known, the performance of the theoretically developed stave volume table is the critical element.

Two groups of trees were used to check the estimates. One group was woods-run trees and the other was made up of specially selected stave-quality trees, so it was felt that any limitations of the theory in practice would be detected. Stave operators recovered all the stave material they could from felled trees and stave volume was measured. Trees were discarded when staves cut from them did not meet standards described previously. The 20 selected stave trees remaining and 21 woods-run trees were analyzed. These trees ranged from 16 to 30 inches d.b.h. The woods-run trees had a merchantable length of one to three logs, while the selected stave trees ranged from one to five logs. Actual volumes recovered were subtracted from estimated volumes. These differences are the errors in estimation that were analyzed to determine the validity of the chord-foot volume table. We have the following pertinent statistics:

Average error[4]	
Chord feet per woods-run tree	−0.075
Chord feet per selected stave tree	+0.150
Standard deviation	
Chord feet per woods-run tree	±5.79
Chord feet per selected stave tree	±7.05
Coefficient of variation for average tree[5]	
Woods-run tree	±13%
Selected stave tree	±14%

[4]Plus differences are overestimates, minus differences are underestimates.

[5]Standard deviation as percent of actual recovery.

It is important to remember that these statistics record error in estimating gross volumes. In light of the standard deviation, the standard error of the mean would amount to approximately ±1.4 chord feet. Average errors for these data, then, are not significant. In addition, the errors were normally distributed. The coefficient of variation, the standard deviation as a percent of actual recovery, is based on operations where maximum stave recovery was the objective. Since it was either ±13 or ±14 percent, depending on whether woods-run or selected stave trees were measured for recovery, 10 to 15 percent of estimated volume seems a fair estimate of the variation to be expected from tree to tree. This variation is about the same as the error found in board-foot volume tables. Further analysis of estimates did not reveal any significant trend in error of estimation due to tree size.

These results show that the theoretical estimates of gross stave volume are sound. Now we can relax assumptions about product specifications, reduce gross volume to net volume, and modify the board-foot/chord-foot ratio.

Different Product Specifications

Product specifications that accept a top diameter other than 10 inches will alter the expected board-foot recovery. Too, lower stave bolt specifications can increase stave recovery. For example, accepting a 4-inch square rather than a 5-inch square enables a producer to split more sections from a bolt. The volume table can be corrected for different product specifications from field data or from local relations already established. A local volume table already published for white oak in Missouri[6] is used to illustrate correction. The 5-split recovery from 1-log trees in Missouri is compared with recovery shown here from 1-log trees (fig. 2, table 4). The comparison is described by the equation:

$$Y = 8.4 + 0.80X$$

where Y equals adjusted values for Missouri and X equals the stave values in table 1.

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[6]Smith, Richard C. Measuring white oak stave bolts. Jour. Forestry 50: 38. 1952.

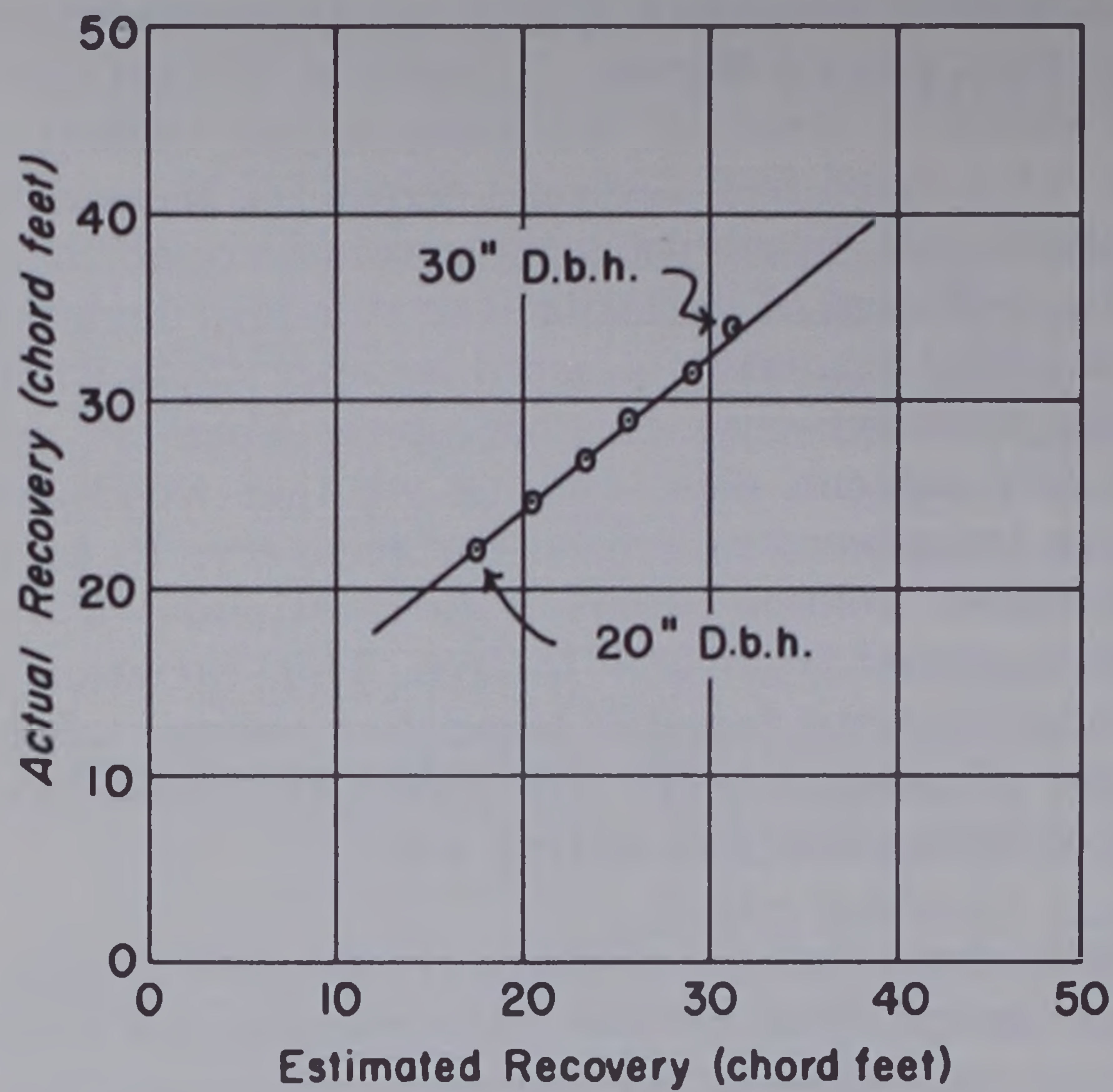

FIGURE 2. — The relation between yield from 1-log cuts as predicted in volume table developed here and yield obtained in a Missouri study.

Volume adjustments can be made for the product specifications of other areas. It is important to correct both board-foot volumes and chord-foot volumes for such local specifications. Without these adjustments serious systematic errors would have resulted in the Missouri example. New local board-foot/chord-foot ratios can be calculated from an adjusted volume table.

TABLE 4. — Chord-foot correction for 1-log trees (in chord feet)

D.b.h. (inches)	Missouri yield from five splits	Yield from alternatives table, 5-inch square	Corrected yield for Missouri
20	22.2	17.5	22.4
22	24.6	20.2	24.6
24	26.9	23.1	26.8
26	29.2	26.0	29.2
28	31.3	28.6	31.3
30	33.9	31.3	33.8

Adjusting for Net Volume

So far we have been dealing with gross volume. Anyone buying or appraising timber, however, is interested only in net volume or the gross volume less cull. The adjustment is made as follows:

$$\frac{\text{Board feet}}{\text{Chord feet}} \times \frac{\text{Percent of board feet recovered as net}}{\text{Percent of chord feet recovered as net}}$$

If we assume a product-product ratio of $\frac{12}{1}$ for gross volumes and assume recoveries of 93 percent and 75 percent respectively, the adjusted ratio becomes:

$$\frac{12}{1} \times \frac{.93}{.75} = \frac{11.16}{.75} = \frac{14.9}{1}$$

This adjusted ratio indicates that a net volume of 14.9 board feet is equivalent to 1 chord foot.

Adjustment for cull can be applied to a table of chord-foot/board-foot ratios (such as found in tables 2 and 3). In the case above, the data in tables 2 and 3 can be adjusted by the application of the factor 1.24 (93 percent divided by 75 percent) to arrive at net board-foot-per-chord-foot relations.

Selecting From Alternate Markets for Whole Trees

Often a timberland owner has clear-cut alternate markets for selling trees. For example, he may have the option of selling either to a sawmill man or a stave producer. He needs to determine what trees should be sold for saw logs and for staves. Ordinarily, a flat stumpage rate is quoted for each product, but sometimes graded stumpage rates are quoted.

We have previously explained volume relations; now price relations need to be considered to determine the break-even point between saw-log and stave-bolt production and the market that is the most profitable.

The break-even point occurs when the gross board-foot/chord-foot ratio (tables 2 and 3) is equal to the ratio of chord-foot price to board-foot price multiplied by the ratio of the net board-foot-recovery percentage to the net stave-bolt-recovery percentage. The equation is as follows:

$$\frac{\text{Product}_1}{\text{Product}_2} = \frac{\text{price of product}_2}{\text{price of product}_1} \times \frac{\text{percent net recovery product}_2}{\text{percent net recovery product}_1}$$

The use of this equation is illustrated below wherein problems are solved on the basis of chord-foot and International ¼-inch rule board-foot measurements.

The problem: *Given flat stumpage rates, what trees suitable for both products should be sold as stave trees and as saw-timber trees? How can we set a price on these trees?*

The data:

Board-foot stumpage rate .... $15.00 per thousand board feet
Net product as percent of gross recovery .... 90 percent

Chord-foot stumpage rate .... $0.40 per chord foot
Net product as percent of gross recovery .... 60 percent

The solution: The price ratio is adjusted for net volume instead of using the gross board-foot/chord-foot ratio. Price ratio is computed as follows:

$$\frac{.40\,(.60)}{.015\,(.90)} = 17.6$$

Only trees whose board-foot/chord-foot ratio exceeds 17.6 should be sold for saw logs. All others should be sold for staves.

The decision: On the basis of volume ratios in table 2, consider sale of only the following trees for saw logs:

16 inches d.b.h. 2- and 3-log trees
18 inches d.b.h. 3- and 4-log trees

Multiply the total volume for each tree by the percent recovered as net and by the price per unit of volume. Use these as tree sale prices.

The problem: *Given graded stumpage rates, what trees suitable for both products should be sold as stave trees and as sawtimber trees? How can we set a price on these trees?*

The data:

Tree grade	Board-foot product		Chord-foot product	
	Value per thousand board feet	Net recovery	Value per chord foot	Net recovery
	Dollars	Percent	Dollars	Percent
1	30.00	90	0.60	70
2	21.00	90	.50	60
3	8.00	90	.30	50

The solution: The price ratio of chord feet to board feet adjusted for cull is computed as follows for each tree grade:

Tree grade	Calculation (chord-foot/board-foot price ratio)	
1	$\frac{.60(.70)}{.030(.90)}$	$\frac{15.6}{1}$
2	$\frac{.50(.60)}{.021(.90)}$	$\frac{15.9}{1}$
3	$\frac{.30(.50)}{.008(.90)}$	$\frac{20.8}{1}$

Grade 1, 2, and 3 trees should be sold for sawtimber when their board-foot/chord-foot ratio exceeds 15.6, 15.9, and 20.8, respectively. All other trees should be sold for staves.

The decision: Consider selling the following trees for saw logs:

Tree grade	D.b.h. for saw-log sale (inches)			
	1-log trees	2-log trees	3-log trees	4-log trees
1	16, 30	16, 18	16, 18	18
2	16	16	16, 18	18
3		16	16	16

Multiply the gross volume for each tree by the percent recovered as net product and by the price per unit of volume. Use these as tree sale prices.

Establishing Values for Multi-Product Trees When Several Markets Exist

Timber sold on a competitive bid is sometimes subcontracted to buyers of special products or made into products for which it was not originally valued. For these reasons, among others, sale prices frequently exceed minimum prices set for bids by a surprising amount. In such situations, the buyer had alternatives for trees or parts of trees that either were not or could not be anticipated by those setting the minimum sale price. White oak trees originally appraised for sawtimber, for example, may be bid in for much more than the appraised price if the competitors for the timber have outlets for staves or veneer or for both. By using our board-foot/chord-foot ratios for volumes of the different products that could be realized, we can place values on individual trees. An example is given below to show how to decide what parts of trees should be appraised for each product and how to place a value on trees with a potential for product combinations.

The problem: *Given flat stumpage rates, how can we both determine the product combinations that will give maximum returns and set a price on these trees?*

The data:

Product	Net as percent of gross	Stumpage rate	Product specifications
Saw logs	90	$15.00 per thousand board feet	all sections 10 inches d.i.b. and larger
Stave bolts	60	$ 0.40 per chord foot	all sections 14 inches d.i.b. and larger
Veneer bolts	40	$50.00 per thousand board feet	all sections 18 inches d.i.b. and larger

The solution:

Alternatives	Calculations	Price ratios
Saw logs or stave bolts	$\frac{.40(.60) \text{ chord feet}}{.015(.90) \text{ board feet (saw logs)}}$	17.6
Saw logs or veneer bolts	$\frac{.05(.40) \text{ board feet (veneer)}}{.015(.90) \text{ board feet (saw logs)}}$	1.5
Stave bolts or veneer bolts	$\frac{.40(.60) \text{ chord feet}}{.05(.40) \text{ board feet (veneer)}}$	12

The price ratios are used to determine the optimum product for each log under different marketing conditions.

Active alternate markets	Deciding the best product for different logs
Saw logs and stave bolts	Appraise all logs having ratios over 17.6 (table 3) for saw logs, all others for stave bolts
Saw logs and veneer bolts	Appraise all logs suitable for both products for veneer bolts
Stave bolts and veneer bolts	Appraise all logs suitable for both products having ratios over 12.0 for veneer
Saw logs, veneer bolts, and stave bolts	Appraise all logs suitable for veneer having a ratio over 12.0 for veneer bolts, appraise all not suitable for veneer having ratios less than 17.6 for stave bolts, and appraise the remainder for saw logs

The decision:

Active alternate markets	Logs to be selected for particular products					
	Veneer bolts		Stave bolts		Saw logs	
	D.b.h.	Log no.	D.b.h.	Log no.	D.b.h.	Log no.
	inches		inches		inches	
Saw logs and stave bolts			(All logs not listed for saw logs)		18 20 22	2,3,4 3,4 4
Saw logs and veneer bolts	(All logs over 18 inches scaling diameter)				(none)	
Stave bolts and veneer bolts	(All logs over 18 inches scaling diameter not listed for stave bolts)		24 26	3,4 3,4		
Stave bolts, veneer bolts, and saw logs	24 26 28 30	1,2 1,2 1,2,3 1,2,3	(All logs not listed)		18 20 22	2,3,4 3,4 4

Multiply the total volume of each log first by the percent of the product recovered as net volume, then by the unit price, and sum the log values to determine tree sale prices.

Some Advantages of this Valuation Procedure

Extension of the reasoning here adopted to other product combinations will value trees under different market conditions. This would replace the value per thousand board feet now currently used. Yields for other products can be related to tree diameters to develop new ratios for use wherever valuation is a problem. The forest administrator would no longer be concerned when timber sold for one use was used for a higher use, since he has developed a value per tree and the buyer can use it for whatever he chooses.

The Central States Forest Experiment Station is headquartered at Columbus, Ohio and maintains major field offices at:

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